Conductance of a Single-mode Electron Waveguide with Statistically Identical Rough Boundaries

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Abstract

Transport characteristics of pure narrow $2D$ conductors, in which the electron scattering is caused by rough side boundaries, have been studied. The conductance of such strips is highly sensitive to the intercorrelation properties of inhomogeneities of the opposite edges. The case with completely correlated statistically identical boundaries (CCB) is a peculiar one. Herein the electron scattering is uniquely due to fluctuations of the asperity slope and is not related to the strip width fluctuations. Owing to this, the electron relaxation lengths, specifically the localization length, depend quite differently on the asperity parameters as compared to the conductors with arbitrarily intercorrelated edges. The method for calculating the dynamical characteristics of the CCB electron waveguides is proposed clear of the restrictions on the asperity height.

72.10.-d; 72.15.Rn; 73.23.-b

I. INTRODUCTION

Application of narrow conducting junctions with extremely small cross dimensions in contemporary microelectronics has generated a great variety of works on transport properties of such conductors. These properties were proved to be substantially controlled by scattering of electrons at random inhomogeneities of the conductor boundaries (see, e.g., Refs. [1–9] and references therein). In particular, in Ref. [2] pure single-mode $2D$ conductors were shown to exhibit all peculiarities characteristic for one-dimensional disordered systems. Their conductance is specified by the coherent electron-surface scattering which causes the localization effects. This certainly constrains lengthwise dimensions of narrow microjunctions in view of the exponential increase of their resistivity upon growing the length.

When producing $2D$ conductors of quite small width it is highly possible, owing to the technology, for the opposite boundaries of the strips to have exactly the same or sufficiently close statistical properties. Among all the models of such statistically identical rough boundaries two substantially different are distinguished. One of them includes the strips with no correlation between the asperities of the opposite edges. Within the other model, correlation between the asperities of the opposite boundaries is just the same as the correlation at any strip edge. Boundaries of the latter type will be referred to as completely correlated (CCB). In Ref. [2] the electron scattering caused by irregularities of only one boundary of the conducting strip was analyzed, the other being perfectly smooth. The obtained results are clearly applicable for the former (not intercorrelated) kind of boundaries. At the same time, the CCB conductors have not received due attention so far.

In this contribution, the CCB case is examined and shown to be the special one. The model considered is physically equivalent to that when the conductor width keeps constant (or nearly constant) along the whole length, despite inhomogeneities of the strip edges. The local mode structure of the electron waveguide remains therein undisturbed. As a result, the electron scattering is due not to the asperity heights, whose values are not restricted in the problem, but to the asperity slopes only.

It is well known that the by-height scattering is controlled by the parameter $(k_F\sigma)^2$ (k_F the Fermi wavenumber of electrons), and the electron relaxation rate is proportional to the square of the r.m.s. asperity height σ (see, e.g., Refs. [2,7]). We argue below that in the single-mode CCB strips the main controlling factor is the ratio $(\sigma/R_c)^4$ (R_c the correlation radius of the boundary asperities). Therefore, the electron scattering rate is proportional to higher, namely the fourth, power of σ. At first glance it should give rise to an increase of the localization length as compared to that from Ref. [2]. However, it is not the case as a rule. In a single-mode CCB strip even with mildly sloping boundary asperities the electron localization length at certain, easily reachable, conditions appears to be much less than the by-height scattering length.

II. FORMULATION OF THE PROBLEM

Let a two-dimensional conducting strip of the length L and the average width d occupy the region of (x, z) plane specified by the inequalities

$$-L/2 \le x \le L/2, \qquad \xi_1(x) \le z \le d + \xi_2(x). \tag{1}$$

The functions $\xi_{1,2}(x)$ describe asperities of the edges of the strip. We assume them continuously differentiable random processes with zero mean values. The correlation properties thereof will be thoroughly discussed below.

In accordance with the standard linear response theory [10] conductance (as well as conductivity) is expressed through product of differences between the advanced and retarded one-electron Green functions (see, e.g., Refs. [11,12]). In what follows the electron scattering will be supposed weak (see Eq. (15)). It is well-proved [13,14] that under these conditions one can neglect the products of the like Green functions (both retarded and both advanced) in the general expression for the conductance. Taking into account the relation between the advanced and retarded Green functions, the conductance $G(L)$ of the strip, divided by the conductance quantum $e^2/\pi\hbar$, at zero temperature is represented as

$$\frac{G(L)}{e^2/\pi\hbar} = -\frac{4}{L^2}\int_{-L/2}^{L/2}dx\int_{\xi_1(x)}^{d+\xi_2(x)}dz\int_{-L/2}^{L/2}dx'\int_{\xi_1(x')}^{d+\xi_2(x')}dz'\frac{\partial\mathcal{G}(x,x';z,z')}{\partial x}\frac{\partial\mathcal{G}^*(x,x';z,z')}{\partial x'}\,. \quad (2)$$

Here $\mathcal{G}(x,x';z,z')$ is the retarded one-electron Green function obeying the equation

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+(k_F+i0)^2\right]\mathcal{G}(x,x';z,z') = \delta(x-x')\delta(z-z') \quad (3)$$

with k_F the Fermi wavenumber. The asterisk in Eq. (2) denotes complex conjugation. We consider the function $\mathcal{G}$ meeting zero Dirichlet boundary conditions at the strip edges $z=\xi_1(x)$ and $z=d+\xi_2(x)$ whereas at the strip ends $x=\pm L/2$ the radiative conditions are satisfied.

In solving problems related to the boundary scattering in waveguides the coordinate transformation is often applied to smooth out both boundaries toward ideally flat (see, e.g., Ref. [7]). For our purpose it is more conveniently to smooth only one side of the strip. Let it be, for definiteness, the lower one which we smooth out to the line $z_{new}=0$. This is done by a transformation of the transverse coordinate, $z_{new}=z_{old}-\xi_1(x)$, accompanied by the corresponding change of the longitudinal velocity operator. As a result, the perturbation $\xi_1(x)$ is transferred to both the conductance expression (2),

$$\begin{aligned}\frac{G(L)}{e^2/\pi\hbar} = &-\frac{4}{L^2}\int_{-L/2}^{L/2}dx\int_0^{d(x)}dz\int_{-L/2}^{L/2}dx'\int_0^{d(x')}dz'\,\times\\ &\times\left[\frac{\partial}{\partial x}-\xi_1'(x)\frac{\partial}{\partial z}\right]\mathcal{G}(x,x';z,z')\left[\frac{\partial}{\partial x'}-\xi_1'(x')\frac{\partial}{\partial z'}\right]\mathcal{G}^*(x,x';z,z')\,,\end{aligned} \quad (4)$$

and to the Green function equation (3) which takes the form

$$\begin{aligned}&\left[\frac{\partial^2}{\partial x^2}+\alpha^2\frac{\partial^2}{\partial z^2}+(k_F+i0)^2\right]\mathcal{G}(x,x';z,z')-\\ &-\left[\hat{\mathcal{U}}(x)\frac{\partial}{\partial z}-\hat{\mathcal{V}}(x)\frac{\partial^2}{\partial z^2}\right]\mathcal{G}(x,x';z,z') = \delta(x-x')\delta(z-z')\ .\end{aligned} \quad (5)$$

From here on we use the notations listed below. In Eq. (4) $d(x)$ stands for the local width of the strip,

$$d(x) = d + \Delta\xi(x), \qquad \Delta\xi(x) = \xi_2(x) - \xi_1(x)\ , \tag{6}$$

with $\Delta\xi(x)$ being the width fluctuation. Next, in Eq. (5) the factor α^2 and the effective zero-mean-valued 'potentials' $\hat{\mathcal{V}}(x)$ and $\hat{\mathcal{U}}(x)$ of the electron-surface interaction have been introduced,

$$\alpha^2 = 1 + \langle {\xi_1'}^2(x)\rangle, \qquad \hat{\mathcal{V}}(x) = {\xi_1'}^2(x) - \langle {\xi_1'}^2(x)\rangle, \qquad \hat{\mathcal{U}}(x) = \xi_1'(x)\frac{\partial}{\partial x} + \frac{\partial}{\partial x}\xi_1'(x). \tag{7}$$

The angular brackets $\langle \ldots \rangle$ denote averaging over realizations of the random functions $\xi_{1,2}(x)$, primes in functions stand for derivatives over their arguments.

To analyze the electron transport in a narrow $2D$ waveguide, where quantization of the electron transverse motion is rather considerable, we apply the discrete, i.e. 'mode', representation in the coordinate z. The Green function now turns to zero at $z = 0$ and $z = d(x)$. Then, allowing for this, we present $\mathcal{G}(x, x'; z, z')$ as a series,

$$\mathcal{G}(x, x'; z, z') = \frac{2}{\sqrt{d(x)d(x')}} \sum_{n,n'=1}^{\infty} G_{nn'}(x, x') \sin\left(\frac{\pi n z}{d(x)}\right) \sin\left(\frac{\pi n' z'}{d(x')}\right) . \tag{8}$$

By substituting Eq. (8) into Eq. (5) we arrive at the following set of equations for the Fourier coefficients $G_{nn'}(x, x')$,

$$\begin{aligned} &\left\{\frac{\partial^2}{\partial x^2} + k_n^2(x) + i0 - \left[\frac{\pi n}{d(x)}\right]^2 \hat{\mathcal{V}}(x)\right\} G_{nn'}(x, x') - \frac{4}{d(x)} \sum_{m=1}^{\infty} A_{nm}\hat{\mathcal{U}}(x) G_{mn'}(x, x') + \\ &+ \frac{2}{d(x)} \sum_{m=1}^{\infty} \hat{\Phi}_{nm}(x) G_{mn'}(x, x') = \delta_{nn'}\delta(x - x')\,. \end{aligned} \tag{9}$$

Here the locally quantized value $k_n(x)$ of the electron longitudinal wavenumber and the coefficient matrix A_{nm} are given by

$$k_n(x) = \left(k_F^2 - \left[\frac{\pi n \alpha}{d(x)}\right]^2\right)^{1/2}, \qquad A_{nm} = \frac{nm}{n^2 - m^2} \sin^2\left[\frac{\pi}{2}(n - m)\right]. \tag{10}$$

We omit the expression for the matrix potential $\hat{\Phi}_{nm}(x)$ in view of its awkwardness. It is only important for us to point out its being the functional of $\xi_1'(x)$ and $\Delta\xi'(x)$ and turning to zero as $\Delta\xi'(x) = 0$.

The equation (9) covers scattering of electrons by rough boundaries of two-dimensional electron waveguide at arbitrary correlation conditions for the asperity heights $\xi_{1,2}(x)$. In this work, our intention is to discuss the case of not arbitrary but statistically identical strip sides. Moreover, we deal with the conductors where asperities of the opposite sides correlate with each other just as they do within every edge of the strip. For stating this CCB model of $2D$ junction we use the correlation equalities

$$\langle \xi_i(x)\rangle = 0; \qquad \langle \xi_i(x)\xi_k(x')\rangle = \sigma^2 \mathcal{W}(x - x'), \qquad i, k = 1, 2. \tag{11}$$

Here $\mathcal{W}(x)$ is the correlation coefficient specified by the unity amplitude and the correlation radius R_c. As a consequence of Eq. (11), the following correlation functions equal zero,

$$\langle \xi_{1,2}(x) \Delta\xi(x') \rangle = \langle \Delta\xi(x) \Delta\xi(x') \rangle = 0 \ . \tag{12}$$

For the weak electron-surface scattering (or gaussian statistics of the asperities), Eq. (12) leads to the same result for any averaged quantity as at $\Delta\xi(x) = \Delta\xi'(x) = 0$. So hereinafter the local width of the strip, $d(x)$, can be replaced by its average value d, and the last term containing the potential $\hat{\Phi}_{nm}(x)$ in l.h.s. of Eq. (9) can be properly dropped. Below we omit the subscript '1' on the function $\xi_1(x)$ for simplicity.

Deviation of the factor α^2 from unity in k_n, Eq. (10), could be significant at 'sharp' asperities as it causes effective decrease of number of the modes propagating in the waveguide. Taking this into account is of no crucial problem. Nevertheless, we introduce one more simplification not to complicate calculations. We will consider only the mildly sloping boundary inhomogeneities for which

$$|\xi'_{1,2}(x)|^2 \ll 1 \ . \tag{13}$$

This allows to put henceforward $\alpha^2 = 1$ and neglect perturbation of the velocity operators in the expression (4) for the conductance.

Note that in Eq. (9) the term containing the potential $\hat{\mathcal{V}}(x)$ describes the intrachannel (intramode) electron scattering with conservation of the quantum number n. At the same time, the perturbation operator $\hat{\mathcal{U}}(x)$ results, to the basic approximation, just in the intermode scattering since the corresponding sum over m in Eq. (9) is free of the term with $m = n$ ($A_{nn} = 0$, in accordance with a definition from Eq. (10)). The inverse lengths of the electron scattering from the potentials $\hat{\mathcal{V}}(x)$ and $\hat{\mathcal{U}}(x)$ are proportional, in the main approximation, to $\langle \xi'^4(x) \rangle$ and $\langle \xi'^2(x) \rangle$, respectively. If the boundary asperities are mildly sloping (13), these lengths could substantially differ. However, in the case of narrow conductors with a single propagating electron mode (the ultra-quantum limit), when

$$1 < k_F d/\pi < 2, \tag{14}$$

the term linear in the operator $\hat{\mathcal{U}}(x)$ multiplied by $G_{11}(x, x')$ is not present in Eq. (9). That is why the spatial decrease of the average single-mode Green function $< G_{11}(x, x') >$ is not determined by the interchannel but the intrachannel electron scattering with the attenuation length proportional to σ^{-4}. This is just the case we analyze below.

For the benefit of our study an important point is to presume the electron-surface scattering weak. That is the electron relaxation length L_1 in the open channel with $n = 1$ has to be large as compared to 'microscopic' lengths of our problem, specifically the electron wavelength k_1^{-1} and the correlation radius R_c. What is more, the conductor length L will be supposed obeying the similar requirements, which are necessary for averaging procedure to be reasonable. All these conditions can be formulated through the inequality

$$\max\{k_1^{-1}, R_c\} \ll \min\{L_1, L\} \, . \tag{15}$$

Note that we do not assume any predetermined interrelation between L and L_1 as well as between k_1^{-1} and R_c.

To get the starting expression for the single-mode conductance $G_1(L)$ one should substitute Eq. (8) into Eq. (4). In line with the weak-scattering conditions (15), all the Green functions with $n, n' \neq 1$ contribute $G_1(L)$ slightly. Then for the dimensionless single-mode conductance $T_1(L)$ we have

$$T_1(L) = \frac{G_1(L)}{e^2/\pi\hbar} = -\frac{4}{L^2} \iint_{-L/2}^{L/2} dx dx' \frac{\partial G_{11}(x,x')}{\partial x} \frac{\partial G_{11}^*(x,x')}{\partial x'}. \tag{16}$$

As it was pointed out, the equation (9) with $n = n' = 1$ does not contain the first degree of the potential $\hat{\mathcal{U}}(x)$ at the function $G_{11}(x,x')$. For this reason in a single-channel strip the electron-surface scattering caused by the potential $\hat{\mathcal{U}}(x)$ manifests itself in higher orders of its magnitude. To obtain the correct equation for $G_{11}(x,x')$ one has to follow the procedure outlined in Appendix A. In the event of mildly sloping asperities (13) and weak-scattering approximation (15) we get

$$\begin{aligned} &\left(\frac{\partial^2}{\partial x^2} + k_1^2 + i0\right) G_{11}(x,x') - \left(\frac{\pi}{d}\right)^2 \hat{\mathcal{V}}(x) G_{11}(x,x') - \\ &- \left(\frac{4}{d}\right)^2 \int_{-L/2}^{L/2} dx_1\, \hat{K}(x,x_1) G_{11}(x_1,x') = \delta(x-x')\,. \end{aligned} \tag{17}$$

Here the novel perturbation operator has occurred with the kernel

$$\hat{K}(x,x') = -\sum_{m=2}^{\infty} A_{1m}^2 \left[\hat{\mathcal{U}}(x) G_m^{(0)}(|x-x'|)\hat{\mathcal{U}}(x') - \langle \hat{\mathcal{U}}(x) G_m^{(0)}(|x-x'|)\hat{\mathcal{U}}(x')\rangle\right]. \tag{18}$$

The unperturbed Green functions $G_m^{(0)}(|x-x'|)$ of the modes $m \geq 2$ attenuate exponentially along the strip over the electron wavelengths,

$$G_m^{(0)}(|x-x'|) = -\frac{1}{2|k_m|} \exp\Big(-|k_m||x-x'|\Big), \qquad |k_m| = \left[(\pi m/d)^2 - k_F^2\right]^{1/2}. \tag{19}$$

Thus, the problem is reduced to calculating the statistical moments $\langle T_1^n(L)\rangle$ of the conductance (16) with the single-mode Green functions found from Eq. (17).

III. TWO-SCALE MODEL

The equation (17) for the Green function $G_{11}(x,x')$ is strictly one-dimensional and, consequently, makes it possible to analyze in detail the effects of coherent multiple scattering of electrons. Inhomogeneities of the strip edges enter now the scattering potentials of the equation rather than the boundary conditions for the Green functions. In accordance with the weak-scattering assumption (15), there exist two groups of substantially different spatial scales in our problem. On the one hand, it is a group of 'macroscopic' lengths, L_1 and L, and on the other a pair of the 'microscopic' lengths, k_1^{-1} and R_c. This suggests that it is reasonable to apply for calculating the Green function G_{11} the two-scale model of oscillations.

Take the well-known representation for the one-dimensional Green function $G_{11}(x,x')$,

$$G_{11}(x,x') = \widetilde{W}^{-1}[\psi_+(x)\psi_-(x')\Theta(x-x') + \psi_+(x')\psi_-(x)\Theta(x'-x)]\,. \tag{20}$$

In Eq. (20), the functions $\psi_\pm(x)$ are the linearly independent solutions of the uniform equation (17) with the radiation conditions satisfied at the strip ends $x = \pm L/2$, respectively. The Wronskian of those functions is $\widetilde{W}$, and $\Theta(x)$ is the Heaviside unit-step function. The

functions $\psi_\pm(x)$ will be sought as superposition of modulated waves propagating in opposite directions along the x-axis,

$$\psi_\pm(x) = \pi_\pm(x)\exp(\pm ik_1x) - i\gamma_\pm(x)\exp(\mp ik_1x)\ . \tag{21}$$

The radiation conditions for the functions $\psi_\pm(x)$ are stated as the 'initial' conditions for the amplitudes $\pi_\pm(x)$ and $\gamma_\pm(x)$, i.e.

$$\pi_\pm(\pm L/2) = 1\ , \qquad\qquad \gamma_\pm(\pm L/2) = 0\ . \tag{22}$$

Emphasize that the amplitudes $\pi_\pm(x)$ and $\gamma_\pm(x)$ in Eqs. (21), (22) are varied at the characteristic length L_1 (or L). Therefore in the framework of two-scale approximation (15) they are smooth functions of x as compared to the rapidly oscillating exponents $\exp(\pm ik_1x)$ and the correlation coefficient $\mathcal{W}(x)$.

According to Eqs. (20), (21), the problem of calculating the Green function G_{11} is reduced to finding the smooth amplitudes $\pi_\pm(x)$ and $\gamma_\pm(x)$. Within the assumption (15), the appropriate equations for them are deduced by the standard method of averaging over the rapid phases (see, e.g., Ref. [15]). For doing that one should substitute $\psi_\pm(x)$ of the form (21) into the uniform equation (17) and multiply it by $\exp(\mp ik_1x)$. Then the equation obtained should be averaged over the spatial interval of a length intermediate between the above introduced macroscopic and microscopic scales. The same should be done using the multiplier $\exp(\pm ik_1x)$. As a result, we get the set of dynamic equations,

$$\begin{aligned} &\pi'_\pm(x) \pm i\eta(x)\pi_\pm(x) \pm \zeta^*_\pm(x)\gamma_\pm(x) = 0\ , \\ &\gamma'_\pm(x) \mp i\eta(x)\gamma_\pm(x) \pm \zeta_\pm(x)\pi_\pm(x) = 0\ . \end{aligned} \tag{23}$$

The variable coefficients $\eta(x)$ and $\zeta_\pm(x)$ are the space-averaged random fields associated with the electron-surface interaction potentials from Eq. (17). The function $\eta(x)$ is a real field whereas $\zeta_\pm(x)$ are the complex conjugated ones. Since our concern is with the quantities averaged over realizations of the random function $\xi(x)$, only the correlation properties of the fields are of decisive importance. In the Appendix B the exact expressions for $\eta(x)$ and $\zeta_\pm(x)$ are written and it is shown that within the two-scale model (15) all these functions can be properly regarded as δ-correlated gaussian random processes with the correlation relations as follows,

$$\begin{aligned} &\langle\eta(x)\rangle = \langle\zeta_\pm(x)\rangle = \langle\eta(x)\zeta_\pm(x')\rangle = \langle\zeta_\pm(x)\zeta_\pm(x')\rangle = 0\ , \\ &\langle\eta(x)\eta(x')\rangle = {L_f}^{-1}\delta(x-x')\ , \quad \langle\zeta_\pm(x)\zeta^*_\pm(x')\rangle = {L_b}^{-1}\delta(x-x')\ . \end{aligned} \tag{24}$$

Here in Eq. (24) two lengths are present, L_f and L_b, specified by the expressions

$$\begin{aligned} {L_f}^{-1} = &\frac{1}{2k_1^2}\left(\frac{\pi\sigma}{d}\right)^4 \int_{-\infty}^{\infty}\frac{dq_x}{2\pi}\, q_x^4 W^2(q_x)\ \times \\ &\times\left\{1 + \frac{8}{\pi^2}\sum_{m'=2}^{\infty} A_{1m'}^2\left[(2k_1+q_x)^2 g_{m'}^{(0)}(k_1+q_x) + (2k_1-q_x)^2 g_{m'}^{(0)}(k_1-q_x)\right]\right\}^2\ , \end{aligned} \tag{25}$$

$$L_b{}^{-1} = \frac{1}{2k_1^2}\left(\frac{\pi\sigma}{d}\right)^4 \int_{-\infty}^{\infty} \frac{dq_x}{2\pi}\,(q_x^2 - k_1^2)^2 W(q_x - k_1) W(q_x + k_1) \times$$
$$\times \left[1 + \left(\frac{4}{\pi}\right)^2 \sum_{m=2}^{\infty} A_{1m}^2 (q_x^2 - k_1^2) g_m^{(0)}(q_x)\right]^2 . \tag{26}$$

The function $W(q_x)$ is the Fourier transform of the correlation coefficient $\mathcal{W}(x)$ from Eq. (11), and $g_m^{(0)}(q_x)$ is the analogous transform of the unperturbed Green function (19),

$$g_m^{(0)}(q_x) = -\frac{1}{q_x^2 + |k_m|^2} . \tag{27}$$

Taking advantage of Eqs. (20), (21), and (24) we can show that superposition of the inverse lengths (25) and (26) is the inverse outgoing length of attenuation of the average Green function $\langle G_{11}(x,x')\rangle$. It is reasonable then to associate this superposition with the length L_1 from Eq. (15), i.e.

$$L_1^{-1} = L_f^{-1} + L_b^{-1} . \tag{28}$$

From the derivation presented in Appendix B, as well as from the appearance itself of the expressions (25) and (26), it is easy to establish that the length L_f is related to the forward electron scattering (i.e. without changing the sign of the velocity x-component) while L_b to the backward scattering. In our consideration the length L_f specifies the correlator $\langle\eta(x)\eta(x')\rangle$ whereas L_b controls the correlator $\langle\zeta_\pm(x)\zeta_\pm^*(x')\rangle$. Hence the conclusion is clear that the fields $\eta(x)$ and $\zeta_\pm(x)$ from Eq. (23) are responsible for the forward and backward electron scattering, respectively.

IV. CONDUCTANCE AND RESISTIVITY MOMENTS

The next step is to express the dimensionless conductance (16) through the smooth amplitudes $\pi_\pm$ and $\gamma_\pm$ and to average it subsequently over the random fields $\eta(x)$ and $\zeta_\pm(x)$. To do this substitute Eqs. (20), (21) into Eq. (16). After a succession of simple transformations with the use of the inequalities (15) we get the formula for the conductance of a single-mode strip,

$$T_1(L) = |\pi_\pm^{-1}(\mp L/2)|^2 . \tag{29}$$

From this equality it naturally follows that the quantity $\pi_\pm^{-1}(\mp L/2)$ can be regarded as the amplitude transmission coefficient of the waveguide of the length L.

Introduce the amplitude reflection coefficient $\Gamma_\pm(x) = \gamma_\pm(x)/\pi_\pm(x)$, in accordance with Eq. (21). From Eq. (23) it can be established that the quantities $\pi_\pm^{-1}(x)$ and $\Gamma_\pm(x)$, in line with their physical meaning, obey the flow conservation law,

$$|\Gamma_\pm(x)|^2 + |\pi_\pm^{-1}(x)|^2 = 1. \tag{30}$$

As a consequence of Eqs. (23), (22), the coefficient $\Gamma_\pm(x)$ satisfies the Riccati-type equation with the homogeneous initial condition,

$$\pm \frac{d\Gamma_\pm(x)}{dx} = 2i\eta(x)\Gamma_\pm(x) + \zeta_\pm^*(x)\Gamma_\pm^2(x) - \zeta_\pm(x) \ , \tag{31}$$

$$\Gamma_\pm(\pm L/2) = 0 \ .$$

Being closed, this equation is more convenient to analyze than the set (23). Therefore, expressing the single-mode conductance (29) through $|\Gamma_\pm(\mp L/2)|^2$ by the use of the conservation law (30), we will perform all the following calculations in terms of the reflection coefficient $\Gamma_\pm(x)$ rather than the transmission one $\pi_\pm^{-1}(x)$.

Attention should be given to the fact that the field $\eta(x)$ may be eliminated from Eq. (31) by concurrent phase transformations of the reflection coefficient $\Gamma_\pm(x)$ and the fields $\zeta_\pm(x)$. These transformations retain the correlation relations (24) for the new renormalized fields $\zeta_\pm(x)$ unaffected. That is one can put the random function $\eta(x)$ in Eq. (31) equal to zero. Consequently, the outcome for arbitrary moment of the conductance is specified by just backscattering of electrons, i.e. by the attenuation length L_b from Eq. (26).

Now let us define the n-th moment of the reflection coefficient squared modulus,

$$R_n^\pm(x) = \langle |\Gamma_\pm(x)|^{2n} \rangle \ . \tag{32}$$

From Eq. (31), one can obtain, basing on the Furutsu-Novikov formula and the correlation relations (24), the differential-difference equation for that moment (see, e.g., Ref. [16]),

$$\pm \frac{dR_n^\pm(x)}{dx} = -\frac{n^2}{L_b}\left[R_{n+1}^\pm(x) - 2R_n^\pm(x) + R_{n-1}^\pm(x)\right], \qquad n = 0, 1, 2, \dots \ , \tag{33}$$

with the initial condition on the coordinate x

$$R_n^\pm(\pm L/2) = \delta_{n0}. \tag{34}$$

Besides the condition (34), we have $R_0^\pm(x) = 1$ and $R_n^\pm(x) \to 0$ as $n \to \infty$, in accordance with the definition (32).

Solution of Eq. (33) that matches all the above conditions can be expressed through the distribution function $P_L^\pm(u, x)$ and, upon due parametrization, represented as

$$R_n^\pm(x) = \int_1^\infty du P_L^\pm(u, x) \left(\frac{u-1}{u+1}\right)^n . \tag{35}$$

In line with this representation, statistical moments of the conductance (29) can be written through the same distribution function,

$$\langle T_1^n(L) \rangle = \langle (1 - |\Gamma_\pm(\mp L/2)|^2)^n \rangle = \int_1^\infty du P_L^\pm(u, \mp L/2) \left(\frac{2}{u+1}\right)^n . \tag{36}$$

So just the probability density $P_L^\pm(u, x)$ is of our need.

Substitute $R_n^\pm(x)$ in the form (35) into equation (33) and perform some elementary transformations. Then we get for $P_L^\pm(u, x)$ the Fokker-Plank equation

$$\pm L_b \frac{\partial P_L^\pm(u, x)}{\partial x} = -\frac{\partial}{\partial u}(u^2 - 1)\frac{\partial P_L^\pm(u, x)}{\partial u}, \tag{37}$$

which is supplemented, according to Eq. (34), by the initial conditions on the coordinate x,

$$P_L^{\pm}(u, \pm L/2) = \delta(u - 1 - 0) \ . \tag{38}$$

From the equality $R_0^{\pm}(x) = 1$ normalization of the function $P_L^{\pm}(u,x)$ to unity follows. In its turn, this implies the distribution function to be integrable over the variable u, in particular, at $u \to 1$ and $u \to \infty$.

The solution of Eq. (37), which satisfies the above pointed requirements, is well-established (see, e.g., Ref. [19]). It can be obtained by the use of the Mehler-Fock transformation [17,18] and found to have the conventional form

$$\begin{aligned} P_L^{\pm}(\cosh\alpha, x) = \frac{1}{\sqrt{8\pi}} \left(\frac{L \mp 2x}{2L_b}\right)^{-3/2} \exp\left(-\frac{L \mp 2x}{8L_b}\right) \times \\ \times \int_{\alpha}^{\infty} \frac{v\,dv}{(\cosh v - \cosh\alpha)^{1/2}} \exp\left[-\frac{v^2}{4}\left(\frac{L \mp 2x}{2L_b}\right)^{-1}\right] , \\ u = \cosh\alpha, \qquad \alpha \geq 0. \end{aligned} \tag{39}$$

With this solution we get from Eq. (36) a relatively simple, as well suitable to analyze, expression for the n-th moment of the dimensionless conductance $T_1(L)$,

$$\begin{aligned} \langle T_1^n(L) \rangle = \frac{4}{\sqrt{\pi}} \left(\frac{L_b}{L}\right)^{3/2} \exp\left(-\frac{L}{4L_b}\right) \times \\ \times \int_0^{\infty} \frac{z dz}{\cosh^{2n-1} z} \exp\left(-z^2 \frac{L_b}{L}\right) \int_0^z dy\, \cosh^{2(n-1)} y \,, \qquad n = 0, \pm1, \pm2, \dots \ . \end{aligned} \tag{40}$$

The formula (40) completely determines the main averaged transport characteristics of a single-mode conducting strip.

V. RESULTS AND DISCUSSION

Let us write down the expressions for the average dimensionless conductance $\langle T_1(L) \rangle$ and resistance $\langle T_1^{-1}(L) \rangle$. Put $n = 1$ in Eq. (40) and take the integrals asymptotically in the parameter L/L_b. Then the asymptotic expressions for the average conductance look like

$$\begin{aligned} \langle T_1(L) \rangle \approx 1 - L/L_b \qquad \text{if} \qquad L/L_b \ll 1 \ , \\ \langle T_1(L) \rangle \approx 2^{-1}\pi^{5/2} (L/L_b)^{-3/2} \exp(-L/4L_b) \qquad \text{if} \qquad L/L_b \gg 1 \ . \end{aligned} \tag{41}$$

At $n = -1$ all the integrals are calculated exactly in Eq. (40), and for the average dimensionless resistance we get the formula,

$$\langle T_1^{-1}(L) \rangle = \frac{1}{2}\left[1 + \exp\left(\frac{2L}{L_b}\right)\right] \ . \tag{42}$$

For the sake of completeness, we also give, without proof, the averaged logarithm of the dimensionless conductance,

$$\langle \ln T_1(L) \rangle = -L/L_b \ . \tag{43}$$

It can be found directly from the equations (23).

The results (41) – (43) match absolutely the concepts of the localization theory for one-dimensional disordered conductors and therefore coincide in appearance with those obtained, in particular, in Ref. [2]. The asymptotics (41) show exponential decrease of the average conductance as the strip length L exceeds the localization length $L_{loc} = 4L_b$. The expression (42) describes exponential growth of the average resistance with growing the strip length L. Needless to say that both the conductance and the resistance are not self-averaged quantities. The main difference of our results from the previously obtained is in the relaxation length L_b, Eq. (26), to be discussed below.

A few words about the validity range for the results (25), (26), (40) – (43). First of all, the boundary asperities of the electron waveguide were supposed to be mildly sloping. The corresponding requirement (13) sets limits on the relation between the asperity height and length,

$$(\sigma/R_c)^2 \ll 1 \ . \tag{44}$$

Additional restrictions result from the weakness of the electron-surface scattering, Eq. (15). In accordance with Eq. (28), the length L_f can be used therein as the parameter L_1, since the inequality $L_f \lesssim L_b$ always holds true. One of the conditions (15), namely, $L_1 \gg R_c$, is reduced to smallness of the Fresnel parameter $k_F\sigma^2/R_c$. In terms of the diffraction theory, it means the absence of the shadowing effect in scattering of the electron waves by rough boundaries (see, e.g., Ref. [20]). This condition can be rewritten via the parameters of our problem as follows,

$$\sigma^2/R_c d \ll 1 \ . \tag{45}$$

The second inequality from Eq. (15), $L_1 \gg k_1^{-1}$, is reduced merely to the product of Eq. (44) and Eq. (45), so it holds automatically. It should be stressed that the requirements of the asperity smoothness, Eq. (44), and the absence of the shadowing effect, Eq. (45), are conventional in solving problems of the wave diffraction at rough surfaces (see, e.g., Ref. [20]). The necessity of using them has not been overcome till now.

It is instructive to note that in solving the diffraction problems the condition of smallness of the so called Rayleigh parameter $(k_z\sigma)^2$ is normally used. In the case of a single-channel strip, Eq. (14), the ratio $(\sigma/d)^2$ plays the role of this parameter. The results presented herein are free of the above restriction. Indeed, the ratio $(\sigma/d)^2$ was not thought to be small at any step of handling the problem. Note that just the statistical identity and complete correlation of the strip edges, Eq. (11), made it feasible to bypass this restriction.

The main result of our work is revealing the remarkable sensitivity of the interference effects in a single-mode waveguide to the intercorrelation properties of the inhomogeneities of the opposite boundaries. To be certain, it is sufficient to compare the localization length L_0, obtained in Ref. [2] for the conducting strip with only one boundary rough, with the length L_b from Eq. (26) of our paper. In the former case $L_0 \propto \sigma^{-2}$, whereas in ours $L_b \propto \sigma^{-4}$. At first glance it would imply the CCB strips to be more transparent for the electrons as against the junctions with arbitrary asperities of the sides. However, it is not the case as a rule. To illustrate this statement, assume the correlation function $\mathcal{W}(x)$ of the asperities

$\xi(x)$ as gaussian, $\mathcal{W}(x) = \exp\left(-x^2/2R_c^2\right)$. Then one can find the lengths L_0 and L_b related to each other as follows,

$$\begin{aligned} L_0/L_b &\sim (\sigma/R_c)^2\,(d/R_c)^2 \qquad \text{if} \qquad R_c/d \ll 1 \ (k_1 R_c \ll 1), \\ L_0/L_b &\sim (\sigma/d)^2 \exp\left(k_1^2 R_c^2\right) \qquad \text{if} \qquad R_c/d \gg 1 \ (k_1 R_c \gg 1). \end{aligned} \tag{46}$$

Note that in Eq. (46) the parameter $(\sigma/d)^2$ should be thought small because the length L_0 was obtained in Ref. [2] under this assumption. It is evident from Eq. (46) that the ratio L_0/L_b in both limiting cases is the product of a small parameter by a large one. The parameters are such that the situation with $L_0 \gg L_b$ is mostly realizable. Indeed, for the small-scale asperities, when $k_1 R_c \ll 1$ $(R_c/d \ll 1)$, this is satisfied if the slope $(\sigma/R_c)^2$ exceeds the small parameter $(R_c/d)^2$. In the case of the large-scale asperities, i.e. $k_1 R_c \gg 1$ $(R_c/d \gg 1)$, the large exponent $(k_1 R_c)^2$ must merely prevail the logarithm $2\ln(d/\sigma)$.

The fact that localization lengths in single-mode strips with different interboundary statistics of the inhomogeneities could deviate significantly from one another can be explained, in our opinion, in a following way. The localization length L_0 from Ref. [2] corresponds to the electron scattering by the effective potential

$$U_1 = \frac{(\pi\hbar/d)^2}{m}\frac{\xi(x)}{d}\ , \tag{47}$$

which depends on just the asperity height $\xi(x)$ (m is the electron mass). In the CCB case, all the scattering potentials contain the gradient $\xi'(x)$ instead of the function $\xi(x)$. Scattering by the potential (47) can be regarded as scattering by the asperity heights (or, what is more precisely, by the waveguide width fluctuations). At the same time, scattering by the potentials from Eq. (17) can be interpreted as caused by the asperity slope fluctuations (or by the waveguide bends). The strength of the by-height and by-slope scattering depends on different parameters. Whereas the scattering from the potential (47) is governed by the Rayleigh parameter $(\sigma/d)^2$, the by-slope scattering depends on the slope parameter $(\sigma/R_c)^2$. Besides, not the least of the factors is the functional dependence of the potentials on the random function $\xi(x)$. Indeed, the potential (47) is linear in $\xi(x)$ whereas the potentials from Eq. (17) are quadratic in $\xi'(x)$. Thus, the distinction between the scattering mechanisms in the waveguide with one boundary rough and in the CCB strip brings the difference of the corresponding relaxation lengths L_0 and L_b.

Another peculiarity of the electron scattering by the strongly correlated identical rough edges is the necessity of taking into account the 'evanescent' waveguide modes, i.e. the non-propagating modes. These modes are present in the last, i.e. the third, term in l.h.s. of the equation (17). As it is evident from the structure of the kernel (18), this term governs intrachannel scattering of the propagating mode with $n = 1$ through interchannel transitions via the virtual evanescent modes with $n \geq 2$. Those transitions contribute to the expressions (25), (26) for the scattering lengths as much, in order of magnitude, as the direct intramode scattering governed by the potential $\hat{\mathcal{V}}(x)$ in Eq. (17). The conclusion immediately follows that neglect of the evanescent modes in solving the problems of waves and particles propagation in waveguides is not quite correct in general. The present results demonstrate that this question needs the special analysis every time it arises.

APPENDIX A: DERIVING THE EQUATION FOR THE SINGLE-MODE GREEN FUNCTION

In the case of the CCB waveguide, when Eqs. (11), (12) hold, the equation (9) for the mode Green function $G_{nn'}(x,x')$ is represented as

$$\left[\frac{\partial^2}{\partial x^2}+k_n^2+i0-\left(\frac{\pi n}{d}\right)^2\hat{\mathcal{V}}(x)\right]G_{nn'}(x,x')-\frac{4}{d}\sum_{m=1}^{\infty}A_{nm}\hat{\mathcal{U}}(x)G_{mn'}(x,x')=\delta_{nn'}\delta(x-x')\,. \tag{A1}$$

This equation with radiative boundary conditions at the strip ends $x=\pm L/2$ is obviously equivalent to the Dyson-type integral equation,

$$G_{nn'}(x,x')=G_n^{(0)}(|x-x'|)\delta_{nn'}+\left(\frac{\pi n}{d}\right)^2\int_{-L/2}^{L/2}dx_1G_n^{(0)}(|x-x_1|)\hat{\mathcal{V}}(x_1)G_{nn'}(x_1,x')+$$
$$+\frac{4}{d}\sum_{m=1}^{\infty}\int_{-L/2}^{L/2}dx_1G_n^{(0)}(|x-x_1|)A_{nm}\hat{\mathcal{U}}(x_1)G_{mn'}(x_1,x'). \tag{A2}$$

Here $G_n^{(0)}(|x-x'|)$ is the unperturbed Green function being the solution of Eq. (A1) at $\hat{\mathcal{V}}(x)\equiv\hat{\mathcal{U}}(x)\equiv 0$.

As $A_{nn}=0$, the equations (A1), (A2) do not contain the terms with $\hat{\mathcal{U}}(x)$ acting on $G_{nn'}(x,x')$. To account for this action we have to substitute $G_{mn'}(x,x')$ in the form (A2) into the last term in l.h.s. of Eq. (A1). In doing so we obtain the perturbative terms proportional to operators $\hat{\mathcal{V}}$, $\hat{\mathcal{U}}\hat{\mathcal{U}}$, and $\hat{\mathcal{U}}\hat{\mathcal{V}}$. Restricting ourselves, in view of the mildly sloping asperities (13), by only the perturbations quadratic in $\xi'(x)$ we neglect the terms containing the product $\hat{\mathcal{U}}\hat{\mathcal{V}}$. Then we get

$$\left(\frac{\partial^2}{\partial x^2}+k_n^2+i0\right)G_{nn'}(x,x')-\left(\frac{\pi n}{d}\right)^2\hat{\mathcal{V}}(x)G_{nn'}(x,x')-$$
$$-\left(\frac{4}{d}\right)^2\sum_{m,m'=1}^{\infty}A_{nm}\hat{\mathcal{U}}(x)\int_{-L/2}^{L/2}dx_1G_m^{(0)}(|x-x_1|)A_{mm'}\hat{\mathcal{U}}(x_1)G_{m'n'}(x_1,x')=$$
$$=\delta_{nn'}\delta(x-x')+\frac{4}{d}A_{nn'}\hat{\mathcal{U}}(x)G_{n'}^{(0)}(|x-x'|). \tag{A3}$$

It immediately follows from Eqs. (A3), (10) that all the off-diagonal Green functions $G_{nn'}(x,x')$ with $n\neq n'$ are small compared to the diagonal ones due to the second term in r.h.s. of Eq. (A3).

Let us rewrite the equation (A3) for the single-mode Green function $G_{11}(x,x')$

$$\left(\frac{\partial^2}{\partial x^2}+k_1^2+i0\right)G_{11}(x,x')-\left(\frac{\pi}{d}\right)^2\hat{\mathcal{V}}(x)G_{11}(x,x')-$$
$$-\left(\frac{4}{d}\right)^2\sum_{m=2}^{\infty}A_{1m}\hat{\mathcal{U}}(x)\int_{-L/2}^{L/2}dx_1G_m^{(0)}(|x-x_1|)A_{m1}\hat{\mathcal{U}}(x_1)G_{11}(x_1,x')-$$
$$-\left(\frac{4}{d}\right)^2\sum_{m,m'=2}^{\infty}A_{1m}\hat{\mathcal{U}}(x)\int_{-L/2}^{L/2}dx_1G_m^{(0)}(|x-x_1|)A_{mm'}\hat{\mathcal{U}}(x_1)G_{m'1}(x_1,x')=\delta(x-x'). \tag{A4}$$

The last term in l.h.s. of this equation has only the off-diagonal Green functions with $m' \geq 2$ and can be consequently omitted. Thus we get from Eq. (A4) the asymptotically justified closed equation for $G_{11}(x,x')$.

The mean value of the perturbative operator quadratic in $\hat{\mathcal{U}}$ in Eq. (A4) differs from zero. The zero-mean-valued operator necessary for the subsequent averaging over the random fields can be obtained by merely subtracting the mean value of the original operator from itself. In doing so we arrive at the equation,

$$\left(\frac{\partial^2}{\partial x^2}+k_1^2+i0\right)G_{11}(x,x')-\left(\frac{\pi}{d}\right)^2\hat{\mathcal{V}}(x)G_{11}(x,x')-\left(\frac{4}{d}\right)^2\int_{-L/2}^{L/2}dx_1\hat{K}(x,x_1)G_{11}(x_1,x')+$$
$$+\left(\frac{4}{d}\right)^2\sum_{m=2}^{\infty}A_{1m}^2\int_{-L/2}^{L/2}dx_1\langle\hat{\mathcal{U}}(x)G_m^{(0)}(|x-x_1|)\hat{\mathcal{U}}(x_1)\rangle G_{11}(x_1,x')=\delta(x-x'). \tag{A5}$$

Here the novel perturbation operator has occurred specified by the kernel $\hat{K}(x,x')$, Eq. (18). Besides, the additional, i.e. the last, term has appeared in l.h.s. of Eq. (A5). The detailed analysis shows that this term gives rise to the small real renormalization of the wavenumber k_1 and takes no effect on the relaxation processes. This permits us to drop it from Eq. (A5) and come directly to the equation (17).

APPENDIX B: FORMULATION OF THE CORRELATION RELATIONS FOR THE SPACE-AVERAGED RANDOM FIELDS

In Sec. III we performed the averaging over the rapid phases and arrived at the equations (23) in which the functions $\eta(x)$ and $\zeta_\pm(x)$ could be written as the sums

$$\eta(x)=S_V^+(x)+S_U^+(x)\,,\qquad \zeta_-(x)=S_V^-(x)+S_U^-(x)\,,\qquad \zeta_+(x)=\zeta_-^*(x)\,. \tag{B1}$$

The random fields $S_V^\pm(x)$ and $S_U^\pm(x)$ are associated with the potentials $\hat{\mathcal{V}}(x)$ and $\hat{K}(x,x_1)$,

$$S_V^\pm(x)=\frac{1}{2k_1}\left(\frac{\pi}{d}\right)^2\int_{x-l}^{x+l}\frac{dx'}{2l}\mathrm{e}^{-ik_1x'}\hat{\mathcal{V}}(x')\mathrm{e}^{\pm ik_1x'}\,, \tag{B2}$$

$$S_U^\pm(x)=\frac{1}{2k_1}\left(\frac{4}{d}\right)^2\int_{x-l}^{x+l}\frac{dx'}{2l}\int_{-L/2}^{L/2}dx_1\mathrm{e}^{-ik_1x'}\hat{K}(x',x_1)\mathrm{e}^{\pm ik_1x_1}\,. \tag{B3}$$

Here the length l is chosen arbitrary within the interval

$$\max\{k_1^{-1},R_c\}\ll l\ll\min\{L_1,L\}\,. \tag{B4}$$

In this Appendix we describe a way to obtain the correlation relations (24). We will demonstrate this with a simple example of correlators of the fields $S_V^\pm(x)$ only. By substituting $\hat{\mathcal{V}}(x)$ in the form (7) into Eq. (B2) and expressing $\xi(x)$ as the Fourier integral, we get

$$S_V^\pm(x)=-\frac{1}{2k_1}\left(\frac{\pi}{d}\right)^2\int_{-\infty}^{\infty}\frac{dq_x}{2\pi}(q_x\mp k_1)\int_{-\infty}^{\infty}\frac{dq_x'}{2\pi}(q_x'-q_x)\exp[i(q_x'-k_1)x]\times$$

$$\times \frac{\sin[(q'_x - k_1)l]}{(q'_x - k_1)l} \left[\tilde{\xi}(q'_x - q_x)\tilde{\xi}(q_x \mp k_1) - \langle \tilde{\xi}(q'_x - q_x)\tilde{\xi}(q_x \mp k_1)\rangle\right] , \tag{B5}$$

with $\tilde{\xi}(q_x)$ being the Fourier transform of $\xi(x)$,

$$\tilde{\xi}(q_x) = \int_{-L/2}^{L/2} dx\, \xi(x) \exp(-iq_x x)\,. \tag{B6}$$

Assuming $\xi(x)$ to be the Gaussian random process we have the correlation equalities for $\tilde{\xi}(q_x)$ resulting immediately from Eq. (11),

$$\langle \tilde{\xi}(q_x)\rangle = 0, \qquad \langle \tilde{\xi}(q_x)\tilde{\xi}(q'_x)\rangle = \sigma^2 W(q_x)\Delta(q_x + q'_x)\,. \tag{B7}$$

Here $\Delta(q_x)$ indicates the 'underlimiting' δ-function,

$$\Delta(q_x) = \int_{-L/2}^{L/2} dx \exp(\pm i q_x x) = \frac{\sin(q_x L/2)}{q_x/2} \to 2\pi\delta(q_x)\,. \tag{B8}$$

From Eqs. (B5) and (B7) we deduce the following integral expression for the binary correlation function

$$\langle S_V^{\pm}(x)S_V^{\pm}(x')\rangle = \left(\frac{1}{2k_1}\right)^2 \left(\frac{\pi\sigma}{d}\right)^4 \int_{-\infty}^{\infty} \frac{dq_x dq'_x dq''_x dq'''_x}{(2\pi)^4} (q_x \mp k_1)(q'_x - q_x)(q''_x \mp k_1)(q'''_x - q''_x) \times$$

$$\times W(q_x \mp k_1)W(q'_x - q_x) \exp[i(q'_x - k_1)x + i(q'''_x - k_1)x'] \frac{\sin[(q'_x - k_1)l]}{(q'_x - k_1)l} \frac{\sin[(q'''_x - k_1)l]}{(q'''_x - k_1)l} \times$$

$$\times \Delta(q'''_x + q'_x \mp 2k_1)\left[\Delta(q''_x + q_x \mp 2k_1) + \Delta(q''_x + q'_x - q_x \mp k_1)\right]. \tag{B9}$$

The integrand of Eq. (B9) contains three types of sharp functions. The first is $\Delta(q_x)$ with variation scale $q_x \sim L^{-1}$, the second, $W(q_x)$, varies at $q_x \sim R_c^{-1}$, and the third-type functions are those of the form $\sin(q_x l)/q_x l$. Owing to Eq. (B4), the function $\Delta(q_x)$ is the sharpest in the integrand. With its aid we take the integrals over q''_x and q'''_x. The q'_x-integral is evaluated through the third-type sharp functions. In such a way we obtain the formula

$$\langle S_V^{\pm}(x)S_V^{\pm}(x')\rangle = \frac{3 \mp 1}{8k_1^2}\left(\frac{\pi\sigma}{d}\right)^4 \int_{-\infty}^{\infty} \frac{dq_x}{2\pi}(q_x \mp k_1)^2 W(q_x \mp k_1) \times$$

$$\times \left[(q_x - k_1)^2 W(q_x - k_1) + (q_x + k_1 \mp 2k_1)^2 W(q_x + k_1 \mp 2k_1)\right] \times$$

$$\times \exp[-i(k_1 \mp k_1)(x + x')] F_l^{\pm}(x - x')\,. \tag{B10}$$

The functions $F_l^{\pm}(x)$ in Eq. (B10) are

$$F_l^{+}(x) = \frac{1 - |x|/2l}{2l}\Theta(2l - |x|)\,, \qquad F_l^{-}(x) = \frac{\sin[4(1 - |x|/2l)k_1 l]}{8k_1 l^2}\Theta(2l - |x|)\,. \tag{B11}$$

The function $F_l^{+}(x)$ is sharp within the scales L_1 and L with mean value equal to unity,

$$\int_{-\infty}^{\infty} dx\, F_l^{+}(x) = 1. \tag{B12}$$

Thus this function can be replaced by the δ-function in the correlator $\langle S_V^+(x)S_V^+(x')\rangle$. At the same time, the function $F_l^-(x)$ is integrally small in the parameter $(k_1 l)^{-2} \ll 1$ and, consequently, is allowed to be put zero. Taking this into account we get the final expressions for the correlators (B9), with the accuracy prescribed by the conditions (B4),

$$\begin{aligned} &\langle S_V^+(x)S_V^+(x')\rangle = L_f^{-1}\{VV\}\delta(x-x')\,; \\ &\langle S_V^-(x)S_V^-(x')\rangle = 0\,. \end{aligned} \tag{B13}$$

Here the notation $L_f\{VV\}$ stands for the electron relaxation length conditioned by the potential $\hat{\mathcal{V}}(x)$ and corresponds to the forward electron scattering. From Eq. (B10) it follows that

$$L_f^{-1}\{VV\} = \frac{1}{2k_1^2}\left(\frac{\pi\sigma}{d}\right)^4 \int_{-\infty}^{\infty} \frac{dq_x}{2\pi} q_x^4 W^2(q_x)\,. \tag{B14}$$

Performing the analogous calculations for the correlators $\langle S_V^{\pm}(x)S_V^{\pm *}(x')\rangle$ we find, with the same accuracy,

$$\begin{aligned} &\langle S_V^+(x)S_V^{+*}(x')\rangle = L_f^{-1}\{VV\}\delta(x-x')\,; \\ &\langle S_V^-(x)S_V^{-*}(x')\rangle = L_b^{-1}\{VV\}\delta(x-x')\,. \end{aligned} \tag{B15}$$

Here $L_b^{-1}\{VV\}$ is the backward-scattering relaxation length specified by the expression

$$L_b^{-1}\{VV\} = \frac{1}{2k_1^2}\left(\frac{\pi\sigma}{d}\right)^4 \int_{-\infty}^{\infty} \frac{dq_x}{2\pi} (k_1^2 - q_x^2)^2 W(k_1 - q_x) W(k_1 + q_x)\,. \tag{B16}$$

Calculation of all the remaining correlators of the functions (B2), (B3), necessary for obtaining the correlation relations for the fields $\eta(x)$ and $\zeta_{\pm}(x)$, can be done in a perfectly similar way. Minor additional complications are connected with the unwieldy structure of the kernel $\hat{K}(x', x_1)$ only, Eq. (18). They can be easily overcome having in mind the weak-scattering conditions (15). The result is given by Eqs. (24) – (26).

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